recurrence problems no limits

1. On the first day of March, a bank loans a man £2500 at a fixed rate of interest of 1.5% per month. This interest is added on the last day of each month and is calculated on the amount due on the first day of the month. He agrees to make repayments on the first day of each subsequent month. Each repayment is £300 except for the smaller final amount which will pay off the loan.

    (a) The amount that he owes at the start of each month is taken to be the amount still owing just after the monthly repayment has been made.

        Let u_n and u_{n+1} represent the amounts that he owes at the start of two successive months. Write down a recurrence relation involving u_{n+1} and u_n. **2**

    (b) Find the date and the amount of the final payment. **4**

Part	Marks	Level	Calc.	Content	Answer	U1 OC4
(a)	2	C	CN	A10, A14	$u_{n+1} = 1{\cdot}015u_n - 300, u_0 = 2500$	2001 P2 Q3
(b)	4	C	CR	A11, A14	1 December, £290·68	

•¹ ic: interpret 1·5% •² ic: state the recurrence relation •³ ss: use recurrence relation •⁴ pd: process •⁵ ic: start final date •⁶ pd: process final payment	•¹ 1·015 *stated or implied by the start of (b)* •² $u_{n+1} = 1{\cdot}015u_n - 300$ and initial value (e.g. $u_0 = 2500$) *stated or implied by the start of (b)* •³ u_1 i.e. £2237·50 •⁴ u_2 **and** u_3 i.e. £1971·06, £1700·63 •⁵ £286·38 •⁶ £290·68 for December payment

2. On the day of his thirteenth birthday, a boy is given a sum of money to invest and instructions not to withdraw any money until after his eighteenth birthday. The money is invested and compound interest of 9% per annum is added after each following birthday. By what percentage will the investment have increased when he withdraws his money just after his eighteenth birthday? **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC4
	4	C	CR	A14	approx. 54%	1991 P1 Q11

•¹ 1.09   •² for using $(...)^5$ •³ approx. 54%

3. The sum of £1000 is placed in an investment account on January 1st and, thereafter, £100 is placed in the account on the first day of each month.

- Interest at the rate of 0·5% per month is credited to the account on the last day of each month.
- This interest is calculated on the amount in the account on the first day of the month.

(a) How much is in the account on June 30th ? **(4)**

(b) On what date does the account first exceed £2000? **(2)**

(c) Find a recurrence relation which describes the amount in the account, explaining your notation carefully. **(3)**

Part	Marks	Level	Calc.	Content	Answer	U1 OC4
(a)	4	C	CR	A10		1997 P2 Q3
(b)	2	C	CR	A10		
(c)	3	C	CR	A14		

(a) $\bullet^1$ 1.005

    $\bullet^2$ £1000 + interest = £1005

    $\bullet^3$ £1005 + £100 + interest = £1110.525

    $\bullet^4$ £1537.93

(b) $\bullet^5$ complete another month

    $\bullet^6$ £2073.94 on Nov.1st

(c) $\bullet^7$ $u_{n+1} = 1.005u_n + 100$

    $\bullet^8$ u_n = amount on 1st day of each month

    $\bullet^9$ $u_0 = 1000$ (on 1st January)

[END OF QUESTIONS]